Chapter 13 (excerpts)

Advanced Implementation of Tables

CS102 Sections 51 and 52

Marc Smith and Jim Ten Eyck

Spring 2007

AVL Trees

- An AVL tree
  - A balanced binary search tree
  - Can be searched almost as efficiently as a minimum-height binary search tree
  - Maintains a height close to the minimum
  - Requires far less work than would be necessary to keep the height exactly equal to the minimum
- Basic strategy of the AVL method
  - After each insertion or deletion
    - Check whether the tree is still balanced
    - If the tree is unbalanced, restore the balance

AVL Trees

- Rotations
  - Restore the balance of a tree
  - Two types
    - Single rotation
    - Double rotation

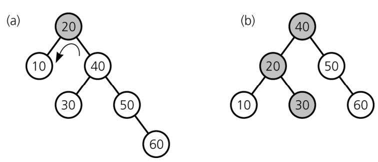

Figure 13-38

a) An unbalanced binary search tree; b) a balanced tree after a single left rotation

AVL Trees

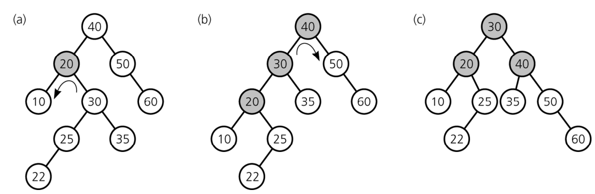

Figure 13-42

a) Before; b) during; and c) after a double rotation

AVL Trees

- Advantage
  - Height of an AVL tree with n nodes is always very close to the theoretical minimum
- Disadvantage
  - An AVL tree implementation of a table is more difficult than other implementations

Hashing

- Hashing
  - Enables access to table items in time that is relatively constant and independent of the items
- Hash function
  - Maps the search key of a table item into a location that will contain the item
- Hash table
  - An array that contains the table items, as assigned by a hash function

Hashing

- A perfect hash function
  - Maps each search key into a unique location of the hash table
  - Possible if all the search keys are known
- Collisions
  - Occur when the hash function maps more than one item into the same array location
- Collision-resolution schemes
  - Assign locations in the hash table to items with different search keys when the items are involved in a collision
- Requirements for a hash function
  - Be easy and fast to compute
  - Place items evenly throughout the hash table

Hash Functions

- It is sufficient for hash functions to operate on integers
- Simple hash functions that operate on positive integers
  - Selecting digits
  - Folding
  - Module arithmetic
- Converting a character string to an integer
  - If the search key is a character string, it can be converted into an integer before the hash function is applied

Resolving Collisions

- Two approaches to collision resolution
  - Approach 1: Open addressing
    - A category of collision resolution schemes that probe for an empty, or open, location in the hash table
      - The sequence of locations that are examined is the probe sequence
    - Linear probing
      - Searches the hash table sequentially, starting from the original location specified by the hash function
      - Possible problem
        - » Primary clustering

Resolving Collisions

- Approach 1: Open addressing (Continued)
  - Quadratic probing
    - Searches the hash table beginning with the original location that the hash function specifies and continues at increments of 1^2, 2^2, 3^2, and so on
    - Possible problem
      - Secondary clustering
  - Double hashing
    - Uses two hash functions
    - Searches the hash table starting from the location that one hash function determines and considers every n^{th} location, where n is determined from a second hash function
- Increasing the size of the hash table
  - The hash function must be applied to every item in the old hash table before the item is placed into the new hash table

Resolving Collisions

- Approach 2: Restructuring the hash table
  - Changes the structure of the hash table so that it can accommodate more than one item in the same location
  - Buckets
    - Each location in the hash table is itself an array called a bucket
  - Separate chaining
    - Each hash table location is a linked list

The Efficiency of Hashing

- An analysis of the average-case efficiency of hashing involves the load factor
  - Load factor α
    - Ratio of the current number of items in the table to the maximum size of the array `table`
    - Measures how full a hash table is
    - Should not exceed 2/3
  - Hashing efficiency for a particular search also depends on whether the search is successful
    - Unsuccessful searches generally require more time than successful searches

The Efficiency of Hashing

- Linear probing
  - Successful search: $\frac{1}{2}[1 + 1(1-\alpha)]$
  - Unsuccessful search: $\frac{1}{2}[1 + 1(1- \alpha)^2]$
- Quadratic probing and double hashing
  - Successful search: $-\log_e(1- \alpha)/ \alpha$
  - Unsuccessful search: $1/(1- \alpha)$
- Separate chaining
  - Insertion is O(1)
  - Retrievals and deletions
    - Successful search: $1 + (\alpha/2)$
    - Unsuccessful search: α

The Efficiency of Hashing

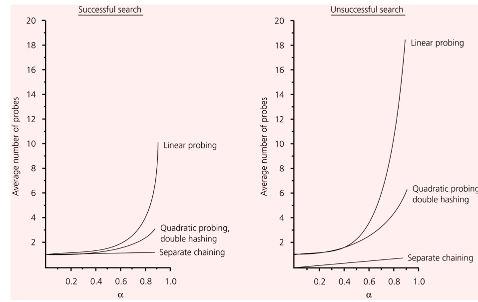

Figure 13-50

The relative efficiency of four collision-resolution methods

What Constitutes a Good Hash Function?

- A good hash function should
  - Be easy and fast to compute
  - Scatter the data evenly throughout the hash table
- Issues to consider with regard to how evenly a hash function scatters the search keys
  - How well does the hash function scatter random data?
  - How well does the hash function scatter nonrandom data?
- General requirements of a hash function
  - The calculation of the hash function should involve the entire search key
  - If a hash function uses module arithmetic, the base should be prime

Table Traversal: An Inefficient Operation Under Hashing

- Hashing as an implementation of the ADT table
  - For many applications, hashing provides the most efficient implementation
  - Hashing is not efficient for
    - Traversal in sorted order
    - Finding the item with the smallest or largest value in its search key
    - Range query
- In external storage, you can simultaneously use
  - A hashing implementation of the `tableRetrieve` operation
  - A search-tree implementation of the ordered operations

The JCF `Hashtable` and `TreeMap` Classes

- JCF Hashtable implements a hash table
  - Maps keys to values
  - Large collection of methods
- JCF TreeMap implements a red-black tree
  - Guarantees O(log n) time for insert, retrieve, remove, and search
  - Large collection of methods

Data With Multiple Organizations

- Many applications require a data organization that simultaneously supports several different data-management tasks
  - Several independent data structures do not support all operations efficiently
  - Interdependent data structures provide a better way to support a multiple organization of data

Summary

- A 2-3 tree and a 2-3-4 tree are variants of a binary search tree in which the balanced is easily maintained
- The insertion and deletion algorithms for a 2-3-4 tree are more efficient than the corresponding algorithms for a 2-3 tree
- A red-black tree is a binary tree representation of a 2-3-4 tree that requires less storage than a 2-3-4 tree
- An AVL tree is a binary search tree that is guaranteed to remain balanced
- Hashing as a table implementation calculates where the data item should be rather than search for it

Summary

- A hash function should be extremely easy to compute and should scatter the search keys evenly throughout the hash table
- A collision occurs when two different search keys hash into the same array location
- Hashing does not efficiently support operations that require the table items to be ordered
- Hashing as a table implementation is simpler and faster than balanced search tree implementations when table operations such as traversal are not important to a particular application
- Several independent organizations can be imposed on a given set of data